Educational standards in mathematics at the end of secondary education - Analysis of students' achievements

Dragana Stanojević[1], Branislav Randjelović[2] and Aleksandra Rosić[3]

[1]Institute for Education Quality and Evaluation, Belgrade, Serbia, [2] University of Niš, Faculty of Electronic Engineering, Serbia, [3]ICT College of applied Studies, Belgrade, Serbia
dstanojevic@ceo.gov.rs, bane@elfak.ni.ac.rs, aleksandra.rosic@its.edu.rs

Standards for general secondary education are based on competencies that will enable students to successfully respond to various life challenges (educational, social, cultural, interpersonal, practical, etc.). Three levels of achievement are defined for each competency - basic, intermediate and advanced. Three levels are cumulative and embedded one in another so that students at the advanced level fulfill requirements from other levels. Methodology for the development of educational standards for the end of secondary education. In this paper we present the results of the empirical examination of students' achievements in Mathematics.

Keywords: Educational standards, Student achievements, General secondary education, Mathematical competences.

1. Educational standards in secondary education in Serbia - context

The education system in the Republic of Serbia consists of three interconnected basic components: curricula, teaching and learning, and evaluation of the achievement of learning outcomes as determined by educational standards. The educational standards are determined on the basis of the general outcomes of education and upbringing - which are the result of the whole process of education. The strength of an educational system is largely reflected in the existence of a developed and efficient system for monitoring and evaluating quality. In a complex system of evaluation of the quality of education, students' achievements have a particularly important place - the knowledge and skills that students acquire during a certain phase of education, represent the quality of "output", and therefore the most important measure of the achievement of the set goals of the educational process (Pešić et al., 2009).

There are some opposing views when introducing educational standards into the education system. One is that standards in education will lead to too much formalism in schools, while the other argues that schools cannot set goals and effectiveness on their own until they fully understand the standards in education (Judo C., 1917).

The educational standards for the end of secondary general education in Serbia have been set for all students in the education system, for those in general secondary education and for those in vocational education. It is far better to choose our own standards that consist of our one values and local socieconomic realities. (Ellman N., 1979)

By developing mathematical competences, it is also necessary for students to develop cross-curricular competences. Thus, the student integrates the different knowledge and skills that he / she has acquired within different subjects. Essentially, working on general and cross-curricular competencies is not competitive with work on content and competencies that are directly related to particular subjects. On the contrary, cross-curricular competences represent a step further in understanding the material and applying the lessons learned, and responsibility for their development rests with all teachers and school subjects. This means that supporting general and cross-curricular competences requires joint planning at the school level, implementation of interactive and active forms of learning, as well as greater autonomy of schools and teachers in implementing educational outcomes. (Rosic, 2015)

The starting points in defining standards, in addition to the general goals and tasks of mathematics, concerned finding real-life examples, linking mathematics to the natural and social sciences as well as its application in solving practical problems. Emphasis is placed on the functionality of the acquired knowledge so that every student is able, at the end of this cycle of education, to identify and understand the role that mathematics plays in the modern world. Every student should perform well-founded mathematical assessments and have knowledge of mathematics to help him / her become a constructive, interested and reflexive citizen.

Mathematics subject standards are based on general subject and subject specific competencies. The general subject competence, within the standard, is to enable the student to think mathematically, to acquire mathematical knowledge and concepts, to critically analyze processes and to improve them, to understand how processes lead to problem solving. The student should develop an exploratory spirit, the ability to think critically, formally and abstractly, as well as deductive and inductive thinking. Furthermore, the student must learn and understand thinking by analogy, develop the ability of mathematical communication and adopt positive attitudes towards mathematics and science in general, apply mathematical knowledge and skills to solve problems in the natural and social sciences and daily life, but also be able to use the acquired knowledge and skills in further education.

Basic level	Defines the level of achievement in certain mathematical competences that student needs to adopt in order to actively and productively participate in different areas of life (social, economic, educational, family, personal, etc.)
Intermerdiate level	Defines the level of achievement in certain mathematical competences that student needs to acquire in order to successfully continues education in various fields
Advanced level	Defines the level of achievement in certain competences that student have to possess in order to be able to successfully continues education at faculty, in area for which those competences are a particularly important requirement.

Table 1: Levels of educational standards (Rosic, 2015)

The general subject competence is divided into three levels: basic, intermediate and advanced. These levels are shown in Table 1. Specific subject competencies are classified into three domains:

Mathematical Knowledge and Reasoning, Application of Mathematical Knowledge and Skills to Problem Solving, and Mathematical Communication. All standards are divided into four major areas: 1) Algebra, 2) Geometry, 3) Sequences, Functions, Derivatives and Integrals, and 4) Combinatorics, Probability, Statistics and Financial Mathematics.

Each individual standard is identified by the code as follows **2.MA.x.y.z.** Number **2** denotes the fact that these are standards for the end of general secondary education and vocational secondary education in the part of general education subjects. **MA** is an abbreviation for the name of the subject, **x** is a level designation (1 - basic, 2 - intermediate and 3 - advanced), **y** is a designation for the area (1 - Algebra, 2 - Geometry, 3 - Sequences, functions, derivatives and integrals and 4 - Combinatorics, Probability, Statistics and Financial Mathematics), and **z** is the number of standards within a given field. For example, code **2.MA.2.1.8.** indicates that this is standard for the end of general secondary education for mathematics, at ntermediate level, in field Algebra and that it is the eighth standard in that field, at that level. Education standards can be roughly divided into sub-areas within the area, shown in Table 2.

Algebra	• Knowledge of different sets of numbers, numbering systems, operations with them, determining the values of numerical expressions and approximate values, • Transformations of algebraic expressions, proving equality and inequality, • Solving equations, inequalities, systems of equations with and without parameters, and • Logic, set operations and relations.
Geometry	• Elementary terms, properties, claims and formulas in planimetry and stereometry, • Isometric transformations, constructions and proofs, • Trigonometry, and • Analytical geometry and vectors.
Sequences, functions, derivatives and integrals	• Sequences and mathematical induction, • Functions and their graphics, • Derivations and • Integrals.
Combinatorics, Probability, Statistics and Financial Mathematics	• Combinatorics, • Probability, • Statistics and • Financial mathematics.

Table 2: Subdomains of mathematics

2. Results of testing of student achievements

In the process of creating educational standards for the subject of mathematics, they have been methodologically developed in two cycles, through the following stages:

- the stage of expert evaluation of the working groups on the content and levels of competences, according to the analysis of existing curricula, the results of previous tests;
- empirical verification phase (paper-pencil test and teacher questionnaire) - pilot and main test of student achievement;
- the synthesis phase of expert judgment with the analysis of empirical research results.

The students had math formulas available on the test paper and were allowed to use calculators.

Standard	Item code	EL	AL	%L	%M	%G	%VET4	%VET3	D	%NR
MA.1.1.1.	M301	1	2	25%	52%	21%	11%	2%	0,43	1%
MA.1.1.2.	M401	1	2	18%	52%	24%	6%	2%	0,47	20%
MA.1.1.3.	M302	1	2	47%	73%	42%	43%	10%	0,36	15%
MA.1.1.5.	M303	1	1	56%	89%	50%	36%	0%	0,42	8%
MA.1.1.5.	M601	1	2	19%	38%	14%	2%	0%	0,54	31%
MA.1.1.6.	M402	1	2	20%	37%	22%	10%	4%	0,39	21%
MA.1.1.7.	M602	1	2	27%	43%	22%	19%	7%	0,39	21%
MA.2.1.1.	M306	2	2	45%	56%	31%	23%	8%	0,29	5%
MA.2.1.2.	M101	2	2	26%	65%	28%	9%	12%	0,51	15%
MA.2.1.2.	M403	1	3	3%	26%	5%	1%	0%	0,48	51%
MA.2.1.3.	M404	2	3	6%	23%	6%	5%	0%	0,34	28%
MA.2.1.4.	M603	2	2	38%	65%	38%	20%	16%	0,47	14%
MA.2.1.5.	M102	2	2	28%	59%	11%	7%	2%	0,53	20%
MA.2.1.6.	M604	2	2	26%	49%	23%	16%	13%	0,48	26%
MA.2.1.8.	M605	2	3	7%	24%	9%	4%	7%	0,30	6%
MA.3.1.1.	M103	3	3	7%	19%	6%	10%	10%	0,33	27%
MA.3.1.1.	M304	3	2	10%	62%	26%	11%	16%	0,45	18%
MA.3.1.3.	M606	3	2	14%	39%	14%	9%	16%	0,36	42%
MA.3.1.5.	M305	3	1	49%	63%	40%	31%	22%	0,20	14%

Table 3: Student achievement in Algebra

Table 3 shows the students achievements during main testing regarding educational standards in Algebra. The table contains the following information, for each item: **Standard** - Educational standard tested by item; **Item code** - item code; **EL** - expected level of achievement; **LA** - level of achievement at testing; **%L** - percentage of high school students with social-linguistic major, who completed the item correctly; **%M** - the percentage of high school students with natural and mathematics major, who completed the item correctly; **%G** - the percentage of high school students who completed the task correctly; **%VET4** - the percentage of school students from four-year vocational secondary schools, who completed the task correctly; **%VET3** - the percentage of school students from four-year vocational secondary schools, who completed the task correctly; **D** – item discriminability expressed through correlation of the task with the overall achievement in a given subject, indicating to what extent the achievement of students who overall had high achievements in a given subject differ from those who overall had low achievements in a given subject; **%NR** - percentage of high school students who did not respond.

The items in the table are arranged in accordance to the levels of educational standards. We can notice that there are some discrepancies in the expected and obtained level of achievement of students in 11 items, out of 19. It is expected that students of high schools with major of mathematics have significantly higher achievements than other students. By analyzing individual items, we can conclude that simplest task for students was M303 (see Figure 1).

Solve the equation.

$x^2 + 12x - 13 = 0$

$x_1 =$ ______ and $x_2 =$ _________

Figure 1: Item M303

The students were familiar with this item from a school context and this is one of the tasks they certainly did see in math classes, so it was expected to achieve high achievement here. The students had the lowest achievement in M404 item (see Figure 2). Although the task is very simple in terms of mathematical operations, and if it is known that the students were able to use the calculator, it seems that they have not sufficiently studied the given questions.

Approximate numbers are given:

$8,71 \le a \le 8,75$ and $b = 5,04 \pm 0,01$.

Fill the table as it started.

	$\underline{p}$	$\overline{p}$
a	8,71	8,75
b		
$a+b$		
$a-b$		

Figure 2: Item M404

In the table we can see that tasks M101 and M403 belong to standard MA.2.1.2. from the middle level, but there is also a clear difference in student achievement. The reason for such a large difference in achievement within one standard may be in the very nature of educational standards, which are broad and encompass different algebra knowledge, skills and skills within one standard.

An analysis of the data, obtained for Algebra, shows that students successfully completed item M302. This item is specific, it is not common in teaching practice, so most students probably saw this type of assignment in mathematics for the first time. Bearing in mind cross-curricular correlations, students had to use the acquired knowledge and skills from physics to solve this task, and applied this in solving this problem (see Figure 3).

Barrel on the picture has dimensions h = 9 dm, D = 6,5 dm и d = 5 dm. The formula for volume of the barrel is:

$$V \approx \frac{\pi h}{12}\left(2D^2 + d^2\right).$$

What is the volume of this barrel ($\pi \approx 3{,}14$)?

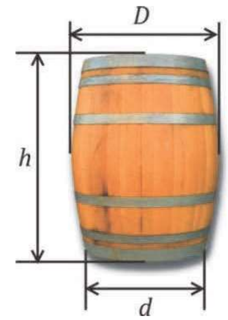

Figure 3: Item M302

Table 4 shows students achievements during main testing of educational standards in Geometry.

Standard	Item code	EL	AL	%L	%M	%G	%VET4	%VET3	D	%NR
MA.1.2.2.	M104	1	1	64%	72%	46%	28%	2%	0,39	7%
MA.1.2.2.	M201	1	3	10%	26%	8%	3%	0%	0,37	22%
MA.1.2.3.	M105	1	3	13%	32%	4%	2%	0%	0,44	38%
MA.2.2.1.	M307	2	3	0%	24%	1%	0%	0%	0,38	44%
MA.2.2.2.	M308	2	2	17%	39%	23%	13%	10%	0,24	31%
MA.2.2.3.	M309	2	2	32%	46%	23%	21%	14%	0,32	13%
MA.2.2.4.	M106	2	3	4%	24%	4%	0%	0%	0,34	33%
MA.2.2.5.	M107	2	3	6%	23%	3%	0%	0%	0,43	42%
MA.3.2.2.	M202	3	3	1%	19%	5%	1%	0%	0,35	38%

Table 4: Student achievement in Geometry

The students had best achievement in item M104, which in its complexity corresponds to the basic level (see Figure 4).

If the surface of the ball is 144π cm^2, how much is its volume?

The volume of the ball is ____________ cm^3.

Figure 4: Item M104

The standard that indicates knowledge of the surface and volume of the ball is also in the list of educational standards for the end of secondary education. However, the achievements of three-year VET students indicates that this issue is very complex for them.

Also, we can see in table that students of natural-mathematical major have had significantly lower achievements, than expected, in items at basic level M201 (see Figure 5) and M105 (see Figure 6).

In the table we can see that tasks M201 and M104 belong to standard MA.1.2.2. from the basic level, but there is also a clear difference in student achievement. The reason for such a large difference in achievement within one standard may be in the very nature of educational standards, which are broad and encompass different mathematical knowledge, skills and skills within one standard.

Draw in equilateral triangle tree line segments so that you get the correct network equilateral three-sided pyramid.

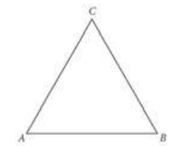

Figure 5: Item M201

The guard spotted a fire from a control tower in the National Park Kopaonik. Height of the tower is 35 m, and the depression angle (the angle under which the person watches the fire in relation to horizontal line) is 22°. How far is the fire from the foot of the tower?

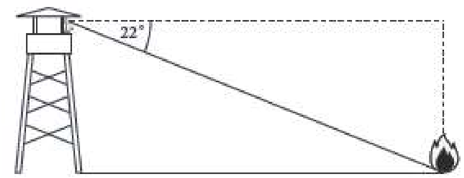

Figure 6: Item M105

These two items are different from the usual school assignments. In the first item, student are expected to estimate the surfaces of geometric bodies in 3D space, while in the second item, student are expected to estimate and calculate distances in a plane, using formulas. Having in mind overall achievements in this field, some secondary analyzes or other researches need to be done to uncover reasons andfactors for low achievement.

Table 5 shows the achievements of students in the area Sequences, functions, derivatives and integrals.

Standard	Item code	EL	AL	%L	%M	%G	%VET4	%VET3	D	%NR
MA.1.3.2.	M501	1	2	9%	45%	15%	3%	0%	0,54	28%
MA.2.3.1.	M405	2	2	32%	56%	33%	26%	22%	0,42	7%
MA.2.3.2.	M502	2	3	5%	26%	0%	1%	0%	0,54	62%
MA.2.3.3.	M503	2	2	24%	49%	40%	15%	10%	0,35	23%
MA.3.3.1.	M406	3	2	14%	44%	21%	7%	4%	0,43	21%
MA.3.3.4.	M407	3	3	4%	16%	0%	1%	0%	0,52	74%
MA.3.4.2.	M505	3	3	0%	4%	1%	1%	0%	0,28	15%

Table 5: Student achievement in Sequences, functions, derivatives and integrals

In the area of Sequences, Functions, Derivatives and Integrals, in 4 of 7 examined standards, there was a coincidence between expected and evaluated level, while for the three standards there were certain disagreements. Standard MA.2.3.1. which was tested by item M405 (see Figure 7) was best solved by the students of natural and mathematical major.

When solving this type of item, students choose various strategies for solving it. One strategy for solve this problem is to form an arithmetic sequence and to calculate largest possible sum of n numbers that is less or equal to 52. Another strategy, which we can call "brutal force", which is alsoused by students, is to count the cards in each row and then adding row by row, until the correct answer is obtained. An interesting fact is that only 7% of students did not even attempt to solve this item.

The picture shows a tower of cards that has three floors. Those with skillful hands can also build multi-storey towers. How many floors can tower maximaly have, using a deck containing 52 cards?

a) The tower may have a maximum of 5 floors.

b) The tower may have a maximum of 6 floors.

v) The tower may have a maximum of 7 floors.

g) The tower may have a maximum of 8 floors.

d) The tower may have a maximum of 9 floors.

Figure 7: Item M405

Standard 3.4.2., tested by item M505, has the worst achievements, ie. only 4 (out of 229) students completed this task correctly, although a large number of students tried to solve it (see Figure 8).

The figure shows the graph of the first derivative of the function $f : [-5,5] \to R$. Fill the line in following sentences.

The function grows at the interval ___________________.

The value of the function is the maximal point ________________.

The function has a maximal point in point ________________.

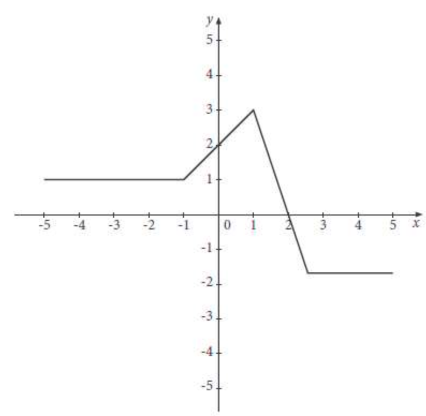

Figure 8: Item M505

Table 6 shows the achievements of students in the field of Combinatorics, Probability, Statistics and Financial Mathematics.

Standard	Item code	EL	AL	%L	%M	%G	%VET4	%VET3	D	%NR
MA.1.5.2.	M203	1	1	66%	86%	55%	48%	35%	0,40	4%
MA.2.5.1.	M204	2	2	31%	47%	28%	27%	31%	0,30	18%
MA.2.5.1.	M205	2	3	28%	35%	20%	16%	14%	0,25	20%
MA.2.5.2.	M504	2	3	6%	36%	12%	2%	0%	0,42	36%

Table 6: Student achievement in Combinatorics, Probability, Statistics and Financial Mathematics

In the field of Combinatorics, Probability, Statistics and Financial Mathematics, half of a total of 47 tested standards, have had matching between expected and diagnosed results, while other half showed different results them expected. Standard MA.1.5.2. tested by item M203 was most successfully solved (see Figure 9).

For six hours of continuosly work, machine packs 1260 boxes of some product. To get 10,000 packaged boxes, you need:

a) more than 12 hours, but less than 24 hours;

b) exactly 24 hours;

v) more than 24 hours, but less than 48 hours;

g) exactly 48 hours;

d) more than 48 hours.

Figure 9: Item M203

To complete this item, students had a lot of different ways to come up with the correct solution.

3. Conclusions and recommendations

A high-quality math program is essential for all students, as it provides them with a solid foundation and great career opportunities, in any of the areas. The examination of student achievement described here, in addition to the many good results it has produced, has shown that students have very little motivation to work when testing is not for assessment. This leads to the fact that individual expectations were well above student achievement. Having common national standards with clearly defined educational outcomes is a step towards greater school accountability but many questions remain. (Lenskaya E., 2013).

The results of this testing and the conclusions reached after the analysis largely influenced the finalization of the proposed educational standards. The data that came from this testing contained information that influenced decision making on how to improve individual standards. Testing of educational standards for the end of secondary general education was the first national testing of students in the final grades of secondary education, and since there were no benchmarks or values before, there was a significant difference in the expected and achieved achievement.

Educational standards are designed to support the basic task of mathematics education, which is to research and develop mathematics teaching at all levels, including foundations, goals and a comprehensive environment (Wittmann, 1995).

No individual or institution like to tell bad news about poor performance on tests. To the extent that this bad news stimulates instructional changes and improvements, however, it can be good new in the long run. (Anrig G., 1985). This should be considered, both when creating new or redesigning existing standards in mathematics, but also when working with students, who must meet the required standards satisfactorily.

Acknowledgment

This work is supported by projects TR32012 and III43007, supported by Ministery of Education, Science and Technological Development of Republic of Serbia.

References

Anrig, G. R. (1985). Educational Standards, Testing, and Equity. *The Phi Delta Kappan,* 66(9), 623-625.

Ellman, N. (1979). Establishing Specific Educational Standards. *Educational Technology,* 19(5), 46-48. Retrieved from http://www.jstor.org/stable/44419078

Judd, C. (1917). EDUCATIONAL STANDARDS. *The Journal of Education,* 85(19 (2129), 507-508.

Lenskaya, E. (2013). Russia's own Common Core. *The Phi Delta Kappan,* 95(2), 76-77.

Pešić, J., Blagdanić, S., Kartal, V. (2009). Defining educational standards for the subject Nature and Society. Quality and efficiency of teaching. Belgrade: Institute for Educational Research

Rosić, A. (2015). General standards of achievement for the end of general secondary and secondary vocational education in the part of general subjects for the subject Mathematics: *1.1. General and cross-curricular competences for the end of secondary education,* Institute for Education Quality and Evaluation

Wittmann, E. C. (1995). Mathematics education as a 'design science'. *Educational studies in Mathematics,* 29(4), 355-374.