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Mathematical Approach to Distant Correlations of Physical Observables and Its Fractal Generalisation

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Abstract: In this paper, the new mathematical correlation of two quantum systems that were initially allowed to interact and then separated is being formulated and analyzed. These correlations are illustrated by many examples and are also connected with fractals at a certain level. The main idea of the paper arises from the EPR paradox, the paradox of Einstein, Podolsky, and Rosen that occurs when the measurement of a physical observable performed on one system has an immediate effect on the other separate system being entangled with it. That is a physical phenomenon, especially when the particles are separated by a large distance. In this paper, we define distant correlations as the advanced method for the exact interpretation of strong connection and influence among those particles even when they are widely separated. On the given topological space (X, τ), we define a notion of τ-metric such that the set X is a τ-metric space and we prove some properties of these spaces. By using this new proposed model, we nullify the contradiction that appears in the EPR paradox. An illustrative example involving fractals is given. This innovative mathematical approach to this physical phenomenon may be attractive for future research in the field of quantum physics.

Keywords: τ-metric; fractals; distant correlations; EPR paradox; quantum physics; topological spaces

1. Introduction

In physics, the principle of locality implies that an object or an event at one position is directly influenced only by its proximate surroundings and therefore it cannot cause an instantaneous action at another point. However, in microscopic physics of quantum entities the result of a measurement on one particle can have a simultaneous effect on another entangled particle, regardless of their distance, which is in the contradiction with the principle of locality and with classical macroscopic physics. This fact is one of the main reasons for the appearance of the famous paper called the EPR paradox by Einstein, Podolsky, and Rosen [1], which implies that quantum physics may not be a complete theory and that maybe there exists some additional unrevealed variable witch integrates both microscopic and macroscopic physic laws. Many scientific papers devoted to the investigation of this subject appeared and large number of various analyzes of the argument have been offered. Among the first scientists who gave the initial inspiration and also pointed out the importance of resolving some contradictions and uncertainties in quantum mechanics was Popper [2]. Fine [3] focuses on Einsteins philosophy of science and establishes his own aspects of realism and antirealism. In addition, Hellman [4] reviewed certain arguments of EPR paradox and clarified some aspects of the "hidden variable" notion. See Krips [5,6] for further consideration of some additional paradoxes that appear in physics on small scales. One of the crucial contributors of understanding the behavior of

quantum particles was Bohr [7] who established the fundamental probabilistic character of quantum measurement. For future investigation, see also Bell [8], Schrodinger [9,10], Einstein [11,12] and many others. Moreover, in last few decades, based on quantum sets [13,14], both fractal and fractional calculus have been deeply exploited in high energy physics. Fractals have been widely used in various applications in science and technology, especially in nanotechnology [15,16]. By using fractal and fractional calculus some of the fundamental physical theories can be explained with considerable smoothness [17,18]. The aim of this paper is to demonstrate that the proposed mathematical model may be applied in quantum physics in order to describe quantum correlations and also to make a connection and possible implementation of the presented results on the conceptive fractal configuration of the spacetime [19,20]. Much research in the field of quantum physics found that the configuration of space has a topological structure and also determined that some of the most fundamental properties of subatomic particles are topological. Topology is of a fundamental importance even to systems in classical mechanics. The first relation between topology and quantum physics was Dirac's [21] attention to resolve the quantization of electric charge. Kauffman and Lomonaco [22] studied the relationship between topological and quantum entanglement. Bakke, Carvalho and Furtado [23] verified the influence of the topology in EPR correlations of cosmic strings. Epperson and Zafiris [24] suggested great importance of implementing topological technics in quantum physics world. They explained a fundamental meaning of mathematical topological laws in quantum particles correlations. In this paper, we discuss a mathematical aspect of the EPR paradox, quantum entanglement and quantum correlations by using special topological spaces with defined metrics and properties not satisfying generally any of the separation axioms T_i, $i = 0, 1, 2$ (for the separation axioms in topological spaces see for example Engelking [25]). The point of the paper is to present (X, τ) topological space with specifically defined properties in order to apply the suggested model to quantum particles relations.

The Characterization of τ-Metric

In this section, we introduce new metrics and analyze topological spaces equipped with such specific metrics in order to apply it in the later sections on particular physical manifestations on small scales. For some basic definitions and concepts of topological spaces, please see [25–30].

Definition 1. *Let X be a topological space, x, y some points in X and $\beta(x), \beta(y)$ their neighbourhood systems, respectively. The relation of density of space in a point we denote with Γ and define as:*

$$x \Gamma y \overset{def}{=} \beta(x) \subset \beta(y). \tag{1}$$

If $x \Gamma y$, we say that the space X has bigger density in the point y than in x.

Obviously, Γ is a partial order relation on X.

If for $x, y \in X$ it holds that $x \Gamma y$, where X is a topological space, then for $A \subset X$ we have that:

$$y \in \overline{A} \text{ implies } x \in \overline{A}, \tag{2}$$

where $\overline{A}$ is the adherence of A ($x \in \overline{A}$ if and only if for every open set U containing x, $U \cap A \neq 0$).

This means that the points that can be comparable according to density are connected in a way that, when ever one of those points is in the adherence, or by the language of physics, in the "influence" of a field of some arbitrary set $A \subset X$, then the other point is also in the influence of the same field. In that sense, the adherence or the influence of the given set, identifies different points as one object in the topological space (X, τ). It means that all topological properties of the space X in the point y are the same as in x.

Definition 2. *Let (X, τ) be a topological space and $\mathcal{A} \subset X \times X$ the set of all ordered pairs of elements $x, y \in X$ which are not comparable in Γ, meaning that:*

$$\mathcal{A} = X \times X - (\Gamma \cup \Gamma^{-1}). \tag{3}$$

The function $p : \mathcal{A} \subset X \times X \to (0, +\infty)$ is τ-metric on X if it holds:

1. *$p(x, y) = p(y, x)$, for every $(x, y) \in \mathcal{A}$*
2. *$p(x, y) \leq p(x, z) + p(z, y)$ for every ordered pairs $(x, y), (x, z), (z, y) \in \mathcal{A}$.*

Remark 1. *For every $x \in X$ is $x\Gamma x$, meaning that $\mathcal{A} \cap \Delta = \varnothing$, so the set $\mathcal{A}$ is a proper subset of $X \times X$. The set $\mathcal{A}$ is nonempty if it contains an ordered pair (x, y) of different elements $x, y \in X$, satisfying that there is at least one neighborhood of x which does not contain y and at least one neighborhood of y which does not contain x. If X is T_1-topological space, we have that $\Gamma = \Gamma^{-1} = \Delta$ and so $\mathcal{A} = X \times X - \Delta$. In this case we can extend τ-metric p in order to obtain metrics on X. Indeed, for every ordered pair $(x, x) \in \Delta$ we put $p(x, x) = 0$. So, τ-metric is nontrivial on a wider class of topological spaces than T_1.*

Example 1. *Every metric function d on a set X provided with topological structure τ, induces τ-metric on X. Indeed, we can consider the function $d|_{\mathcal{A}} = p$. Obviously, this function satisfies all conditions of τ-metric.*

Example 2. *Let us consider the left cone topology on the set of all real numbers R. Open sets are the set R and the collection of all open left cones, i.e., the intervals $(-\infty, a), a \in R$. It is easy to see that this collection is a topology. For every $x, y \in R$ such that $x \leq y$, holds that $\beta(y) \subset \beta(x)$, and so we have that $x\Gamma y$. Therefore, every two elements of the set R are comparable in Γ and $\mathcal{A} = \varnothing$.*

Proposition 1. *In every topological space X holds that:*

$$(for\ all\ x)(for\ all\ y)(\overline{\{x\}} \subset \overline{\{y\}}\ if\ and\ only\ if\ \beta(x) \subset \beta(y)).$$

Proof. Let $\overline{\{x\}} \subset \overline{\{y\}}$ and $U_x \in \beta(x)$. Since $x \in \overline{\{x\}} \subset \overline{\{y\}}$, we have that $x \in \overline{\{y\}}$ and therefore $y \in U_x$, which means that $U_x \in \beta(y)$. We conclude that $\beta(x) \subset \beta(y)$. Assume that $\beta(x) \subset \beta(y)$ and $z \in \overline{\{x\}}$. Every $U_z \in \beta(z)$ contains the point x and so $U_z \in \beta(x) \subset \beta(y)$. Therefore $U_z \in \beta(y)$ and so $y \in U_z$, which leads to $z \in \overline{\{y\}}$. $\square$

Remark 2. *In Definition 1, the condition $x\Gamma y$ can be replaced with the equivalent condition $\beta(x) \subset \beta(y)$.*

Definition 3. *p-open ball denoted by $K_p(x, \epsilon)$, with the center $x \in X$ and radius $\epsilon > 0$ we define as:*
$$K_p(x, \epsilon) = \{y | (y, x) \in \mathcal{A} \text{ and } p(x, y) < \epsilon.\} \tag{4}$$

Remark 3. *Since for every $x \in X$ holds that $x\Gamma x$, the center of each p-open ball $K_p(x, \epsilon)$ is not the element of the ball.*

Example 3. *Let X be an arbitrary set containing more than one point and M a nonempty subset of X with more than one element, such that $X - M \neq \varnothing$. Define $Int A = A \cap M$ for every proper subset $A \subset X$ and let $Int X = X$. The interior operator defined this way satisfies convenient properties and defines a topology τ on X where open sets are X and all subsets of M. The set $M \times M$ is the set of all ordered pairs of elements from X which are incomparable in Γ. Indeed, for every two elements $x, y \in X - M$ we have that $\beta(x) = \beta(y)$ and so it holds that $x\Gamma y$ and $y\Gamma x$. So, every two elements from $X - M$ are comparable in Γ. Let $x \in M$ and $y \in X - M$. Since every neighborhood of y is at the same time the neighborhood of x, we have that $y\Gamma x$. Finally, every two different elements of M are incomparable in Γ due to the fact that topology on M is discrete. Now,*

assume that there is a defined metric d on X. The restriction $p = d|_{M \times M - \Delta}$ is τ-metric on X. If $x \in X - M$, then $K_p(x, \epsilon) = \emptyset$. If $x \in M$, then $K_p(x, \epsilon) = K(x, \epsilon) \cap (M - \{x\})$.

As well as in metric spaces, we may formulate the following.

Definition 4. *Let (X, τ) be a topological space and p a τ-metric on X. p-neibourhood of a point $x \in X$ is every set $U_p(x)$ which contains x and at least one p-open ball centered in x. So, $U_p(x)$ is a p-neibourhood of $x \in X$, if there is $\epsilon > 0$ such that $\{x\} \cup K_p(x, \epsilon) \subset U_p(x)$ holds.*

Definition 5. *Let (X, τ) be a topological space and p a τ-metric on X. A p-open set is a set $U \subset X$ which is a p-neibourhood of all its points.*

Theorem 1. *Let (X, τ) be a topological space, p a τ-metric on X and U a p-open set. Then:*

$$U = \cup_{x \in U} [\{x\} \cup K_p(x, \epsilon_x)]. \tag{5}$$

Proof. Indeed, for every $x \in U$, there is a p-open ball $K_p(x, \epsilon_x)$ such that $K_p(x, \epsilon_x) \subset U$. Therefore, we have that:

$$U = \cup_{x \in U} \{x\} \subset \cup_{x \in U} [\{x\} \cup K_p(x, \epsilon_x)] \subset U,$$

which leads to the conclusion.　□

Theorem 2. *Let (X, τ) be a topological space and p a τ-metric on X. The collection τ_p of p-open sets satisfies:*
(a) If $U_1, U_2 \in \tau_p$, then $U_1 \cap U_2 \in \tau_p$,
(b) If $U_i \in \tau_p$, $i \in I$, then $\cup_i U_i \in \tau_p$.

Proof. Let $U_1, U_2 \in \tau_p$ and $x \in U_1 \cap U_2$. Since U_1 is p-open, we have that $K_p(x, \epsilon_1) \subset U_1$. Similarly, it holds that $K_p(x, \epsilon_2) \subset U_2$. Let $\epsilon = min(\epsilon_1, \epsilon_2)$. Obviously, $K_p(x, \epsilon) \subset U_1 \cap U_2$. Since $U_1 \cap U_2$ is a neighborhood of all its points, we conclude that it is p-open. (Using the induction we may prove the theorem for any finite number of p-open sets. Since $\cap_{i \in \emptyset} U_i = \{x \mid (for\ all\ i)(i \in \emptyset \Rightarrow x \in U_i\} = X$ we conclude that X is a p-open set.)

Now, let $x \in U = \cup_i U_i$, where U_i, $i \in I$, are p-open. There is an index $i \in I$, such that $x \in U_i$, and a p-open ball $K_p(x, \epsilon) \subset U_i \subset U$. The fact that x is arbitrary concludes the proof. (In the case that U is an empty set, for example if $I = \emptyset$, it is p-open set in metric space X. Indeed, since $\cup_{i \in \emptyset} U_i = \{x \mid (\exists i)(i \in \emptyset \wedge x \in U_i)\} = \emptyset$ we conclude that $\emptyset$ is a p-open set.)　□

Theorem 3. *Let (X, τ) be a topological space, p a τ-metric on X and Δ the collection of all subsets of X which contains only mutually incomparable elements in Γ. The collection $\tau_p |_{\widetilde{X}} = \{U \cap \widetilde{X} \mid U \in \tau_p\}$ is a topology on $\widetilde{X}$ induced with p, where $\widetilde{X}$ is any maximal element in Δ regarding the inclusion order.*

Proof. The collection Δ is obviously ordered by inclusion where every chain has an upper bound. So, according to Kuratowski-Zorn Lemma [25], there is a maximal element $\widetilde{X}$ in some maximal chain. Let us prove that $\tau_p |_{\widetilde{X}} = \{U \cap \widetilde{X} \mid U \in \tau_p\}$ is a topology on $\widetilde{X}$ induced by the τ-metric p, where the basis is $\{K_p(x, \epsilon_x) \cap \widetilde{X}, x \in \widetilde{X}\}$.

To prove that, let us choose some $y \in K_p(x, \epsilon_x) \cap \widetilde{X}$. The points x and y are incomparable in Γ and it holds:

$$K_p(y, \epsilon_y) \cap \widetilde{X} \subset K_p(x, \epsilon_x) \cap \widetilde{X}, \tag{6}$$

where $\epsilon_y = \epsilon_x - p(x,y)$. Indeed, if $z \in K_p(y,\epsilon_y) \cap \widetilde{X}$, then z is incomparable with x, so there is $p(x,z)$. Since the triangle relation $p(x,z) \leq p(x,y) + p(y,z)$ for the τ-metric p holds, we have that $z \in K_p(x,\epsilon_x) \cap \widetilde{X}$. It holds:

$$(\{x\} \cup K_p(x,\epsilon_x)) \cap \widetilde{X} \in \tau_p \mid_{\widetilde{X}} . \tag{7}$$

According to (7) and Theorem 3, we conclude that $\tau_p \mid_{\widetilde{X}}$ is a topology on $\widetilde{X}$. □

Remark 4. *Generally, the collection τ_p of p-open sets is not always a topology on X. This is because $\{x\} \cup K_p(x,\epsilon_x) \notin \tau_p$.*

2. The Theoretical Experiment

In the previous section the distant correlations in the form of τ-metric are defined and mathematically formalized in order to understand the actual paradoxical behaviour of the "entangled" particles when they are separated at a large distance. The notion of quantum entanglement is at the core of the discrepancy among classical and quantum physics. For example, it appears naturally when two particles are created at the same point and instant in space. To clarify the central idea of this research we illustrate the well known EPR paradox as the theoretical experiment, that occurs when measurement of a physical observable performed on one system has an immediate effect (the measure could be predicted) on the other separate system being entangled with it. This is a typical example of the entangled particles that are being separated.

The experiment can be interpreted by using electron-positron pairs. Let us assume that there is a certain source which emits electron-positron pairs, such that the electron is sent to the position A and the positron sent to position B. As it is known in quantum mechanics, we may organize our source so that each emitted pair occupies a quantum state called a spin singlet. In this case we say that these particles are entangled. This can be recognized as a quantum superposition of two states, which we call state I and state II. In state I, the electron has spin pointing upward along the z-axis $(+z)$ and the positron has spin pointing downward along the z-axis $(-z)$. On the other hand, in state II, the electron has spin $-z$ and the positron has spin $+z$. The fact is, that it is impossible without measuring to know the definite state of spin of either particle in the spin singlet (It refers to a system in which all electrons are paired, that is, whose overall spin quantum number $s = 0$.) Along z-axis we can obtain one of two possible outcomes: $+z$ or $-z$. Suppose we get $+z$. Consequently, the quantum state of the system collapses into state I. In this particular case, subsequent measurement of the spin along the z-axis in the destination B will be $-z$ with probability 1. There are numerous scientific articles that more briefly investigate various versions of this experiment. The paradox is in the simultaneous synchronized behaviour of the separated entangled particles even at extremely large distances, which contradicts the Einstein relativity theory. Please see for example [31,32].

In the next section, we intend to demonstrate the possible applications of the proposed τ-metric consolidated and defined in the relevant topological space in which we will observe and discuss the entangled particles. This approach will hopefully provide more clear and understandable perception of the behaviour of the electrons in the singlet in the formal mathematical sense.

3. Results and Discussion

3.1. The Distant Correlations and the EPR-Paradox

Let X be some set containing electrons as physical objects having the defined topological structure equipped with the half-metric defined as in Definition 2. Let us observe a pair of electrons from the space X that we consider as one physical object. At this point, we need to precise once again in what sense we use the term "one object"; we say that two electrons present the same physical object if it holds: whenever one of them is in the field of influence of some source, then it is the other one, too. So, two electrons present the same

object if one of the electrons interact with any source from the space *X*, then the other one also interacts with the same source and vice versa. This interpretation is strictly topological (note that the degree of the interaction need not to be the same for both electrons for the reason that it depends of the distance from the source and not from the properties of the field). So, in topological terms, two electrons from the set with the topological structure present the same object if those electrons have the same neighborhood systems. The source of influence described in the EPR paradox "sees" the entangled particles as one object. It seems quite clear that metric defined by (2) is accordant with the behavior of the electrons from the experiment.

Note that it is quite clear that such space cannot be metrizable, meaning that it does not exist any metrics which induces the defined topology. Indeed, assume that such metrics exists, then the topological space *X* had to be T_1 so that every set of points with the same neighborhood system in the space would be reduced only to exactly one point. Consequently, there would exist not one pair of electrons that we might consider as one object in the sense that we explained in details previously. (At this point it is crucial to remind the reader of the fact that every metric space is at least T_1 topological space.)

In the usual settings, we use conventionally certain notion of distance in order to measure the length between physical objects and generally we assume that the observed space is a metric space with the defined metrics (distance) between points. Commonly, we use Hilbert spaces, where the usual distance between points is being defined. However, the main idea of this paper is that, on a micro-level this usual metric does not exists.

Therefore, we formalized certain type of "distance" on the given topological space among physical objects belonging to the set *X*, such that the distance among the objects that are having the same neighbourhood systems is not defined and does not exist, and among every other pair of physical objects in the set *X*, it does exist, it is symmetrical, and it satisfies the triangle relation, according to the Definition 2. If we apply this onto EPR paradox, we can say that the distance among particles which are under the influence of the same source (that are in the adherence of the same set) does not exist, and among particles that are under different influence sources exists (among particles that belong to the different adherence sets) according to the Definition 2. Thus, in the context of the EPR paradox experiment, this "half-metric" does not recognize the distance between the pair of objects (in this particular case particles in the singlet) which are both the field of the same source of influence. In that sense, the adherence or the influence of the given set, identifies different points as one object in the topological space and thus, according to the definition of the τ-metric, the distance among them does not exist. Conclusively, this is a mathematically quite precise form for overcoming the EPR paradox. In addition, note in the context of EPR paradox that, if there appear some new source of influence which differs from the initial one, that interacts with one electron from singlet, then the behavior of entangled electrons stop being connected and simultaneous, which is also directly follows from the definition of τ-metric. Indeed, the distance among the electron pair appears in this case, since they are not anymore the same object in topological sense, meaning they don't have the same neighborhood systems, meaning they do not belong to the same source of influence. This proposed model is not paradoxical and does not contradicts the Einstein relativity theory.

In next paragraph, we illustrate some ideas at micro-level that can be applied in certain domains involving fractals.

3.2. Fractal Application

Quantum mechanics has a fascinating bond with fractals. In fact, on the level of atoms and electrons, there are regular patterns that do not support fractal laws. However, when adding a magnetic field the formation of the electrons' positions catches a fractal form called Hofstadter's butterfly [33]. Electrons in the fractal configuration overtake a fractal design with a dimension of about 1.58. That means that their energy levels manage to split up from a continuous set (electrons have all energies within some scale) to one with clusters. This fact opens a whole new line of analysis and it opens different questions

such as for example what it actually means for electrons to be constricted in non-integer dimensions? Fractals have already established an important role in numerous applications and these results may lead to a great impact on research at the quantum scale. See also fractal behavior of the electrons under the influence of the magnetic field of topological constructions mentioned in previous subsection and the reference [34].

Since the main idea of the paper is to propose the distant relation solution of certain physical appearances, it is of fundamental importance to emphasize the connection between metric spaces and fractals. Generally, we use the Euclidian classical geometry, where the usual distance between points is being defined which describes smooth ideal shapes in the nature. However, there are shapes in the nature more complicated than the normal Euclidean forms, so we need fractal generalization and fractal topology and measure in order to characterize those shapes. Many scientific papers devoted to fractal scaling in Space appeared recently, such as [35].

For future analysis and implementations in macrophysics, possibly the most interesting characteristic of fractals which should be observed thoroughly is the self-similarity. Indeed, the essence of the electron-positron behavior in the EPR paradox is self-similarity. Finally, for the reason of the topological aspect of this research, we give next example which shows explicit fractal application of the proposed topological space on micro-level and with the defined $\tau-$ metric.

Example 4 (Cantor set). *Cantor sets can be obtained geometrically by a removal of a portion (basic Cantor set is obtained by removing the middle one-third set) of the closed unit interval [0, 1] infinitely. The set of points remained in the unit interval after this removal process is over is called the Cantor set. The dimension of such a set is not an integer value. In fact, it has a fractional dimension, making it by definition a fractal. The Cantor set is an example of an uncountable set with measure zero and has potential applications in various branches of mathematics such as topology, measure theory, dynamical systems and fractal geometry. It is also an example of a perfect, nowhere dense subset of $\mathbb{R}$. The Cantor set, firstly configured strictly under mathematical investigation, recently has become almost perfect model for a nonlinear dynamical systems to the distribution of galaxies in the Universe. Indeed, it finds famous place in mathematical analysis and its applications, e.g., see Devaney [36], Hutchinsion [37], Schoenfeld and Gruenhage [38]. Let us recall that the Cantor set is the prototype of a fractal.*

Theorem 4. *Any subset E of the real line R which is compact, totally disconnected and perfect is homeomorphic to the Cantor middle one-third set.*

Since we indicated that the homeomorphic objects can be considered as equal, the proposed metric can be applied obviously on fractals. We will refer now to the research paper raised and analyzed by Naschie [39]. He regarded the space-time as a Cantor set with a very high dimensionality, based entirely on topology and geometry which he called Cantorian. In fact, this is a proper example of the implementation of our mathematical model on this micro-level space-time. Indeed, Cantor set is a topological set. Next, according to Remark 1 and the Example 1, the τ-metric can be induced on such space. Thus, we obtain clear illustration of our results discussed in the paper. Further, it is interesting to analyze the results from [39] which proves that partially ordered sets (posets) can be drawn to model space-time and may be used advantageously in high energy space-time physics, which coincides with the obtained main results. In addition, both in [39,40] the authors present a substantially new approach to quantum gravity and particle physics based on the idea that space-time is basically a large infinite-dimensional but hierarchical, disconnected and thus non-differentiable Cantor Set, which presents pure concretization of our general results proposing topological spaces on micro-level equipped with the τ-metric. Cantor sets are at the heart of modern mathematics, but have never been used explicitly to model space-time in physics, until [39,40] and similar papers therein.

4. Conclusions

In this paper, we introduced a special form of partial metric spaces, researched their properties and suggested some applications in certain domains of physics. The idea for this model appeared naturally from the remarkably increased interaction between mathematics and quantum physics. Especially in the last few decades, many studies devoted to the topology applications in physics occurred. As the result, many mixed concepts were developed, such as for example, topological quantum field theory [41,42]. In accordance and parallel with this fact, the use of fractals increased rapidly in the domain of quantum physics: it is proved that if fractality of the Brownian trajectories [43,44] leads to standard quantum mechanics, then the fractality of the Levy paths [45] leads towards fractional quantum mechanics. Further, the tempered stable isotropic Levy processes, also called relativistic stable processes and its "generalized fractionality properties" lead towards relativistic quantum mechanics in some sense. There are a lot of references on them, such as [46–48]. The fractional quantum mechanics has been developed trough the new fractional path integrals approach. A fractional generalization of the Schrodinger equation [49] has been discovered. The new relationship between the energy and the momentum of the nonrelativistic fractional quantum-mechanical particle has been established. A fractional generalization of the Heisenberg uncertainty relation has been found [50]. In [51], the authors developed and analyzed a new fractal–fractional operational matrix for orthonormal normalized ultraspherical polynomials that have significant importance in physics. At the end, we illustrated the obtained topological results with the fractal example, that mathematically describes the nature of matter and space on quantum level. In addition, see [52,53] for more additional illustrative examples describing fractal employment in physics on both micro and macro level and [54] for the topological and dynamical fractal systems with finitely many maps such as affine or projective, including the role of contractive functions on the existence of an attractor (as the illustration, see Figure 1, obtained as the result of applying a fractal homeomorphism and constructed by using affine transformations, from [54]).
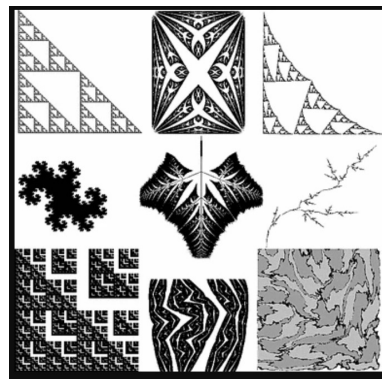

Figure 1. Fractal homeomorphisms.

However, in spite of the strong efforts in developing one comprehensive mathematical model which includes all the aspects of quantum theory, the definite axiomatization of quantum topology still has not been attained. The idea of the paper is to step towards that direction and to provide potentially attractive model to researchers of the entangled particles phenomenon. Although diverse according to the starting viewpoints, rigor mathematics with the universal properties of its structures and an intuitive and experimental physics in mutual collaboration produced most surprising and inspirational results which gave rich ground yet to be explored.

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