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Optimising the thermal environment and the ampacity of underground power cables using the gravitational search algorithm

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Abstract: This study takes into consideration a problem that commonly occurs at the design stage of underground cable lines and that relates to the manner of determining the optimal values for dimensions of cable trench and bedding, interaxial spacings between power cables in flat formation and cable ampacities. The problem is formulated as a non-linear optimisation problem with constraints. The gravitational search algorithm, simulated annealing algorithm and generalised pattern search algorithm are used to solve the problem. The optimal solutions are obtained based on the following three optimisation criteria: (i) minimisation of the total installation costs, i.e. the total costs for producing the cable trench, laying the bedding material, laying the backfill material and installing the cables, (ii) maximisation of the cable ampacity and (iii) a simultaneous application of the minimisation of the total installation costs and the maximisation of the cable ampacity. The constraints on design and control variables are introduced through penalty factors by means of which the objective function is expanded. The procedure is conducted taking into account the effect of drying out of the soil surrounding the cables.

1 Introduction

Based on the heat balance equation for cables [1, 2], it is evident that the cable ampacity can be increased by reducing the soil thermal resistivity which opposes the heat transfer by conduction between the cables and the native soil, as well as between the cables and the ground surface. The application of bedding materials having a lower thermal resistivity than the native soil is one of the traditional manners to control the thermal environment around the cables. It was found that even a small amount of cable bedding material leads to a significant increase in the cable ampacity [3–5]. The effectiveness of the bedding materials becomes particularly evident when the thermal resistivity of the native soil is quite high, or when the drying out of the native soil takes place at lower temperatures [5–7].

It is obvious that underground cable lines could carry higher ampacities if the cable bedding has larger dimensions [4, 8]. This would, however, imply higher total costs for installation of an underground cable line [9]. In addition, the soil zone intended for laying power cables in urban areas is limited [8]. Consequently, the interaxial spacings between power cables affecting the ampacity are also limited. Therefore, there are a number of parameters which directly or indirectly affect the total installation costs and the cable ampacity and whose values can be optimised.

Optimisation of the parameters of underground cable lines does not represent a new problem. Williams *et al.* [10] studied the effect of controlled beddings on the cable ampacity. Minimisation of the total costs for installation of a cable line subject to a specified lower bound on the cable ampacity was discussed in [11]. Moreover, the optimisation procedure was carried out using a non-linear programming formulation [11]. El-Kady first discussed the case with one cable in the bedding. Furthermore, the same optimisation procedure was generalised by El-Kady [11] to the case of several cables in the bedding, taking into account different objective functions. The effect of drying out of the soil surrounding the cables was ignored in [11].

A procedure to determine the ampacities of underground cable lines based on the finite-element method, the concept of efficient electricity transmission and the selection of adequate solution domain was presented in [8]. Although this procedure represents a perfect basis for optimisation of cable line parameters, it lacks an optimisation algorithm and requires the introduction of economic aspects. A method for computing the external thermal resistance of underground cables using the finite-element method was presented in [4]. With this method and without taking optimisation into account, de León and Anders performed a parametric study on how cable ampacity is affected by different sizes and shapes of the beddings. In accordance with [12], the problem of the optimal spatial arrangement of underground power cables in flat formation was solved using the finite-element method and an analytical calculation. The effect of cable bedding on the ampacity was ignored in [12].

Optimisation of the thermal environment and the ampacity of underground power cables in flat formation using the momentum-type particle swarm optimisation method was performed by Ocłoń *et al.* [13]. In this paper, this optimisation problem is solved by means of the gravitational search algorithm (GSA) [14], simulated annealing algorithm (SAA) [15] and generalised pattern search algorithm (GPSA) [16]. The GSA is found to be more stable than the SAA and GPSA. In addition, better results are obtained with the use of the GSA. Moreover, the GSA has proven to be a highly effective tool for solving the various optimisation problems in electric power systems [17–19]. The application of this algorithm enables the determination of optimal solutions for different objective functions, taking into consideration all technical and economic constraints which are typical for underground cable lines. The procedure can include various variables related to different arrangements of power cables in the bedding.

This paper also shows how the application of the GSA, SAA or GPSA with minimum investment funds for installation of an underground cable line can provide required ampacity for a specific type of cables in flat formation with respect to all thermal and physical constraints. The algorithms are applied to a cable system of 35 kV so that the optimal solution is obtained for the three different criteria. The proposed procedure can be applied at the design stage of new underground cable lines or at the redesign stage of existing ones. Additionally, it can be noticed that the GSA, SAA and GPSA are used for the first time for the optimisation of this kind.

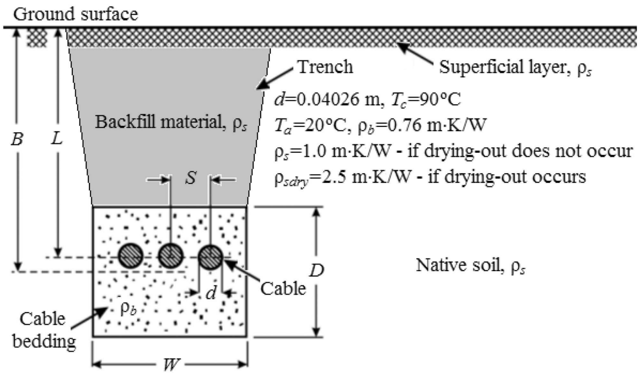

Fig. 1 *Typical underground cable line*

Table 1 Costs of some civil engineering works that are typical for the laying of cables [3]

Civil engineering work	Cost	Cost term
removal of existing asphalt pavement	13.5 $/m^2	$W + 0.6$[a]
excavation and disposal of soil	47.0 $/m^3	$W \times (B + D/2)$
installation of the cable bedding	23.5 $/m^3	$W \times B$
repaving with asphalt	17.8 $/m^2	$W + 0.6$[a]

[a]where an extra width of 0.3 m is added to the width of the cable trench on each side of the cable line route for removal of existing asphalt pavement and repaving with asphalt.

2 Problem formulation

In practice, underground power cables are mainly installed in trefoil or flat formations. The selection of the formation depends on several factors such as earthing of metal screens, cross-sectional area of conductors and available space for installing of cables [20]. A typical cable installation in a trench with cables in flat formation is shown in Fig. 1. It is assumed that the trench is filled with bedding and backfill materials and that the native soil and the backfill material have the same thermal resistivity. All the variables relevant to the optimisation problem are also indicated in Fig. 1. The considerations discussed in this paper are considered to be of a general character and apply to different types of cables installed in various trefoil and flat formations.

It is also possible that there is a layer of material, close to the ground surface, with a thermal resistivity that is different from that for the bedding material and the native soil. This superficial layer can be a protective material, asphalt or soil which dries out due to exposure to solar radiation and has a higher thermal resistivity [6]. In order to simplify the problem, it is also assumed that the superficial layer has the same value of thermal resistivity as the native soil and the backfill material, so that there are only two different materials surrounding the cables.

2.1 Definition of objective function and variables

A general optimisation problem can be described mathematically as follows [21]:

$$\min F(x, u) \tag{1}$$

$$\boldsymbol{g}(x, u) = 0 \tag{2}$$

$$\boldsymbol{h}(x, u) \le 0 \tag{3}$$

where $F(x, u)$ is an objective function, $\boldsymbol{x}$ is a vector of design variables, $\boldsymbol{u}$ is a vector of control variables, $\boldsymbol{g}(x, u)$ is a vector composed of equality constraints and $\boldsymbol{h}(x, u)$ is a vector composed of inequality constraints.

The vector of control variables, whose values will be optimised using the GSA, SAA or GPSA, is defined as

$$\boldsymbol{u} = [W \quad D \quad B \quad S]^{\mathrm{T}} \tag{4}$$

where according to Fig. 1, W is the width of the cable bedding/trench, D is the height of the cable bedding, B is the depth of the cable bedding centre and S is the axial spacing between cables. All these vector elements, i.e. control variables are expressed in metres.

The total costs for installation of an underground cable line C and the cable ampacity I can be changed by varying each control variable. Thus, the total installation costs and the cable ampacity will be considered as design variables, i.e. elements of the following vector:

$$\boldsymbol{x} = [C \quad I]^{\mathrm{T}} \tag{5}$$

Table 1 shows the civil engineering works that are typical for the laying of cables and costs that correspond to them [3].

Based on the data given in Table 1, a total installation cost function $C = C(W, D, B)$ may be formulated as

$$C = 31.3 \times (W + 0.6) + 47 \times W \times (B + D) \tag{6}$$

in $/m, where W, D and B are in metres. The cost of installation of the cable bedding, i.e. the costs associated with the supply, transport and installation of the bedding material amounting to $23.5 \times W \times D$ $/m are included in the cost function (6). The total costs for installation of a cable line (6) are based on research conducted by El-Kady in 1982 [3] and were in force in the suburbs of the north-eastern USA. However, these costs do not differ significantly from the today's total installation costs for the regions of the USA [9, 22].

The cable ampacity I is a complex function of the aforementioned control variables. It is calculated using the following equation of the IEC 60287-1-1 Standard for underground power cables where partial drying-out of the soil occurs [1] (see (7)) where $\Delta\theta$ is the permissible temperature rise of the cable conductor above the ambient/soil temperature in K or °C, W_{d} is the dielectric losses per unit length per phase in W/m, T_1 is the thermal resistance per core between the cable conductor and metal screen in K × m/W, $T_2 = 0$ K × m/W is the thermal resistance between the metal screen and armour, T_3 is the thermal resistance of the polyvinyl-chloride (PVC) outer sheath in K × m/W, T_4 is the thermal resistance of the surrounding soil in K × m/W, n is the number of conductors in the cable, $v = \rho_{\mathrm{sdry}}/\rho_{\mathrm{s}}$ is the ratio of the thermal resistivities of the dry and moist soil zones, $\Delta\theta_x$ is the temperature rise of the boundary between the dry and moist soil zones above the soil temperature (i.e. the critical temperature rise of the soil) in K or °C, R_{ac} is the ac resistance of the cable conductor at its maximum operating temperature in Ω/m, λ_1 is the ratio of the total losses in the metal screen to the total conductor losses and $\lambda_2 = 0$ is the ratio of the total losses in the armour to the total conductor losses.

The values of the control variables fall between the corresponding lower and upper bounds, which are presented in Table 2. The upper bounds are identical to those from [5] and are selected arbitrarily, while the lower bounds depend on cable parameters and the following requirements for underground cable lines: (i) depth of laying L, which shall be equal to 1 m for 35 kV cables; (ii) minimum axial spacing between cables, which shall be equal to the outer diameter of cables d; (iii) distances between the axes of two outer cables and the lateral sides of the bedding/trench closest to them, which shall be not less than 0.15 m [23]; (iv) height of the bedding-part below the cables, which shall be not less

$$I = \left[\frac{\Delta\theta - W_{\mathrm{d}} \times [0.5 \times T_1 + n \times (T_2 + T_3 + v \times T_4)] + (v - 1) \times \Delta\theta_x}{R_{\mathrm{ac}} \times [T_1 + n \times (1 + \lambda_1) \times T_2 + n \times (1 + \lambda_1 + \lambda_2) \times (T_3 + v \times T_4)]}\right]^{0.5} \tag{7}$$

than 0.075 m [23]; and (v) height of the bedding-part above the cables, which shall be not less than 0.17 m [23].

The requirements (iii), (iv) and (v) are not defined by any international standard such as IEC. However, it should be noticed that the requirements are defined in most national standards relating to the installation of underground cables. For instance, in the Australian Network Standard NW000-S0006 [24] or in the Indian Standard IS:1255-1983 [25].

This paper considers the following three optimisation problems:

Problem 1: Minimisation of the total installation costs

$$\text{Minimise} \quad F_1 = C(W, D, B) \text{ objective function} \tag{8}$$

$$\text{subject to} \quad I \geq I(W, D, B, S, L, \rho_\text{b}, \rho_\text{s}(\rho_\text{sdry})) \text{ inequality constraint} \tag{9}$$

and other constraints of the same kind, which will be explained in more details later. In the constraint (9), I represents an upper bound for the cable ampacity, which should be specified in advance (by the user or design engineer) and in accordance with the requirements.

Problem 2: Maximisation of the cable ampacity

$$\text{Minimise} \quad F_2 = -I(W, D, B, S, L, \rho_\text{b}, \rho_\text{s}(\rho_\text{sdry})) \text{ objective function} \tag{10}$$

$$\text{subject to} \quad C \leq C(W, D, B) \text{ inequality constraint} \tag{11}$$

and other constraints of the same kind. In the inequality constraint (11), C represents a lower bound on the budget which is available for cable trench production. It should be specified in advance by the user. Moreover, it should be noted that minimisation of $-I$ is equivalent to maximisation of I.

Problem 3: Simultaneous application of the minimisation of the total installation costs and the maximisation of the cable ampacity

$$\text{Minimise} \quad F_3 = w_C \times F_1 + w_I \times F_2 \text{ unconstrained objective function,} \tag{12}$$

where w_C is the weighting factor for the function C and w_I is the weighting factor for the function I [26]. Since C and I are functions of the same order of magnitude, it is not necessary to carry out normalisation of them [26].

2.2 Inequality constraints

The inequality constraints (3) are defined in the following manner:

(i) *Constraint on the width of the cable bedding:* Based on the fact that the left and right edges of the cable bedding must be positioned at least 0.15 m from the centres of two outer cables in flat formation, this constraint can be expressed as

$$W \geq 2 \times 0.15 + 2 \times S \tag{13}$$

(ii) *Height-to-width constraint:* A rectangular cable bedding with dimensions x and y can be modelled by the following equivalent radius [27]:

$$r_\text{b} = \exp\left[\frac{1}{2} \times \frac{x}{y} \times \left(\frac{4}{\pi} - \frac{x}{y}\right) \times \ln\left(1 + \frac{y^2}{x^2}\right) + \ln\left(\frac{x}{2}\right)\right] \tag{14}$$

where $x = \min(W, D)$ and $y = \max(W, D)$. Equation (14) is only applicable for $y/x \leq 3$. Whenever this is the case, it shall be

$$y \leq 3 \times x \tag{15}$$

In addition to this constraint, the following condition should also be met:

$$r_b > B \tag{16}$$

(iii) *Constraint on the depth of the cable bedding centre:* Based on the fact that the top and bottom edges of the bedding must be positioned at least 0.17 and 0.075 m, respectively, from the outer surfaces of cables in flat formation, this constraint can be expressed as

$$L + 0.075 + \frac{d}{2} - \frac{D}{2} < B < L - 0.17 - \frac{d}{2} + \frac{D}{2} \tag{17}$$

In the particular case when the cable trench is filled with the bedding material to the level of the ground surface, this constraint becomes

$$B \geq \frac{D}{2} \tag{18}$$

(iv) *Constraint on the axial spacing between cables:* With respect to the fact that the axial spacing between adjacent cables in flat formation cannot be less than the outer diameter of cables, the following shall apply:

$$S \geq d \tag{19}$$

2.3 Expanded objective function

The constraints described by expressions (9) and (11), as well as other inequality constraints of the type (3), are taken into account through penalty factors by means of which the objective function F is expanded in the following manner:

$$F_\text{e} = F + p \times \sum_{i=1}^{q} \left| x_i - x_i^\text{lim} \right| \tag{20}$$

where F_e is the expanded objective function to be minimised, F is the function representing the objective function F_1, F_2 or F_3, p is the corresponding penalty factor, q is the number of inequality constraints and x_i^lim is an upper or lower bound on design variable x_i. The bound x_i^lim is defined by

$$x_i^\text{lim} = x_i^\text{max} \text{ if } x_i > x_i^\text{max} \text{ and } x_i^\text{lim} = x_i^\text{min} \text{ if } x_i < x_i^\text{min} \tag{21}$$

where x_i^max and x_i^min are the upper and lower bounds on design variable x_i, respectively, which may be defined as constants or by expressions.

The value of the penalty factor p is determined by the user in accordance with the type of optimisation problem. The penalty factor of 10 is selected for the constraints (9) and (11) that are defined in the optimisation problems 1 and 2. For all other inequality constraints related to these two optimisation problems, it is assumed that p equals 200. In the case of the problem 3, the penalty factor p equals 500 for all the inequality constraints, with the exception of the constraint (18) where the penalty factor is $p =$ 50000.

3 Test example and flowchart of the GSA

In order to optimise the thermal environment and the ampacity of underground power cables, the authors varied the main parameters of the GSA [14]. Based on the results of analysing the three optimisation problems, the following combination of the main parameters of the GSA is suggested: (i) population size $N = 600$, (ii) total number of iterations $J = 400$, (iii) initial gravitational constant $G_0 = 100$ and (iv) user-specified constant $\alpha = 20$. The same values of N and J are used for the SAA and GPSA as well.

This study applies the GSA-, SAA- and GPSA-based optimisations to a standard 35 kV cable, which is produced in compliance with the IEC 60502-2 Standard [28]. Fig. 2 shows the cross-section of this cable. According to [28], the conductor and metal screen of this single-core cable have the rated cross-sections $S_\text{c} = 95\,\text{mm}^2$ and $S_\text{s} = 16\,\text{mm}^2$, respectively. Also, the conductor and metal screen can be loaded continuously to the temperatures $T_\text{c} = 90$ and $T_\text{s} = 70^\circ\text{C}$, respectively. In addition to this, it is assumed that the metal screens are bonded and earthed at both ends.

Table 2 Lower and upper bounds on control variables

Control variable	Lower bound, m	Upper bound, m
W	0.38052	5
D	0.28526	5
B	0.5476	3.5
S	0.04026	2

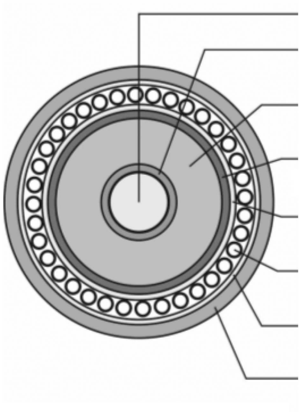

Fig. 2 *Dimensions of the construction elements of a standard 35 kV cable*

In accordance with [23–25, 27], the following dimensions are chosen for the cable trench and bedding: 0.07 m – for the side-by-side spacing between cables in flat formation, 0.1 m – for the height of the bedding-part below the cables, 0.3 m – for height of the bedding-part above the cables and 0.15 m – for the distances between the axes of two outer cables and the lateral sides of the bedding/trench closest to them. Based on these four dimensions, as well as Figs. 1 and 2, it is possible to calculate the control variables W, D, B and S. Accordingly, this gives $W = 0.52052$, $D = 0.44026$, $B = 0.9$ and $S = 0.11026\,\text{m}$.

The total installation costs C and the cable ampacity I, which correspond to the above control variables and also to the ambient conditions defined in Fig. 1, are, respectively, equal to 66.88 $/m and 327.06 A. This test example applies, of course, to the case when the underground cable line is operating under normal conditions.

The flowchart of the GSA for optimisation of the thermal environment and the ampacity of underground power cables is created using [1, 8, 14, 27, 29] and presented in Fig. 3.

4 Calculation results

In all three problems, optimisations are carried out in order to find the best combination of the control variables W, D, B and S. For the purposes of these optimisations, the following parameters are assumed to be constant: depth of laying ($L = 1\,\text{m}$), thermal resistivity of the native soil under normal conditions ($\rho_s = 1\,\text{m} \times \text{K/W}$), thermal resistivity of the native soil in the dry-out state ($\rho_{\text{sdry}} = 2.5\,\text{m} \times \text{K/W}$) and soil temperature ($T_a = 20^\circ\text{C}$). The values quoted in brackets are taken from [29].

It is also assumed that the cable bedding is composed of a mixture of sand and gravel in a weight ratio of 1:1 with 5% fine-grained aggregates of the particles 0–0.063 m [27]. Such cable bedding achieves thermal resistivity of $\rho_{\text{sdry}} = 0.76\,\text{m} \times \text{K/W}$ when it is dried out. The cable ampacity calculations are carried out with the load factor $m = 1$ (i.e. 100% load factor).

Problem 1 – minimisation of the total installation costs: Table 3 presents the optimal values of control variables W, D, B and S, and the design variable C obtained by the minimisation of the objective function (8) for different specified values of the design variable I. The following cable ampacities are taken into consideration: 310, 360 and 410 A.

Problem 2 – maximisation of the cable ampacity: Table 4 presents the optimal values of control variables W, D, B and S, and the design variable I obtained by the maximisation of the objective function (10) for different values of the design variable C. The following total installation costs are taken into consideration: 65, 75 and 85 $/m.

A comparison of convergence profiles of the GSA, SAA and GPSA in the case of the maximisation of the cable ampacity for $C = 75\,\$/\text{m}$ is presented in Fig. 4.

Problem 3 – simultaneous application of the minimisation of the total installation costs and the maximisation of the cable ampacity: Table 5 presents the optimal values of control variables W, D, B and S, and the design variables C and I obtained by the maximisation of the unconstrained objective function (12) for different values of the weighting factors w_C and w_I. The weighting factors are selected with relative values to suggest that: F_1 is twice as important as F_2 ($w_C = 2$ and $w_I = 1$), equal importance is given to F_1 and F_2 ($w_C = 1$ and $w_I = 1$) and F_2 is twice as important as F_1 ($w_C = 1$ and $w_I = 2$).

It should be noted that the successive execution of the GSA gives the values of the objective function which differ slightly from each other (usually in the third digit after the decimal point). The same applies to the values of the control variables. These differences are most evident in the case of the third optimisation problem. This is due to a very high sensitivity of the cable ampacity to variations of the control variables and the specified constraints. In principle, there are a large number of combinations of W, D, B and S that correspond to a single cable ampacity. So, the GSA does not always converge towards the global optimum; however, the differences are small. In order to reduce these differences, higher values for the population size N and the total number of iterations J were selected. This has in turn led to an increase in the time required for the execution of the GSA.

A similar discrepancy was also observed for the SAA and GPSA in the case of the third optimisation problem, but to a much greater extent. Accordingly, the GSA has proven to be the most stable during multiple successive executions, always giving the same or optimal solution which slightly differs from the one obtained in the previous execution of the algorithm. Moreover, it should be noted that the authors have tried to solve the three optimisation problems by means of other algorithms such as the genetic algorithm (GA) or multi-objective GA (MOGA). However, taking very large values of penalty factors into consideration, the GA and MOGA have shown instability, as well as insensitivity to specified constraints. Finally, these two algorithms have not yielded satisfactory results.

5 Discussions

Based on the results presented in Tables 3–5, it can be concluded that the GSA, SAA and GPSA, within the limits allowed by the budget, select values for D and B which correspond to the case when the cable trench is completely filled with bedding material. The conclusion is consistent with the observations reported in [3, 5, 8].

In addition, the algorithms tend to allocate the cables along the bottom edge of the bedding, i.e. to adjust the distance between the cables and the bottom surface of the bedding so that it matches the minimum value of 0.075 m as closely as possible. This means that the cable ampacity is mainly affected by the height D of the cable bedding, which also complies with the conclusions drawn in [3, 5, 8]. This can be explained by the fact that the ground surface behaves as a cooler of cables, which is positioned at a distance $L - d/2$ from them. So, if D is greater than $L - d/2$, the thermal resistivity between the cables and the ground surface has decreased and hence there is an excellent heat-conducting path to the ground surface.

When the previous condition is met, the algorithms will enlarge the width of the cable bedding and the axial spacing between cables in order to increase the cable ampacity. The manner in which this is done is also in compliance with the limits imposed by the budget for the total installation costs C. This behaviour is evident for the solutions obtained using the GSA and SAA (Tables 3 and 4). Compared with the SAA, the GSA gives slightly better results. This means that for the same value of costs, the ampacities obtained using the GSA are slightly higher than the

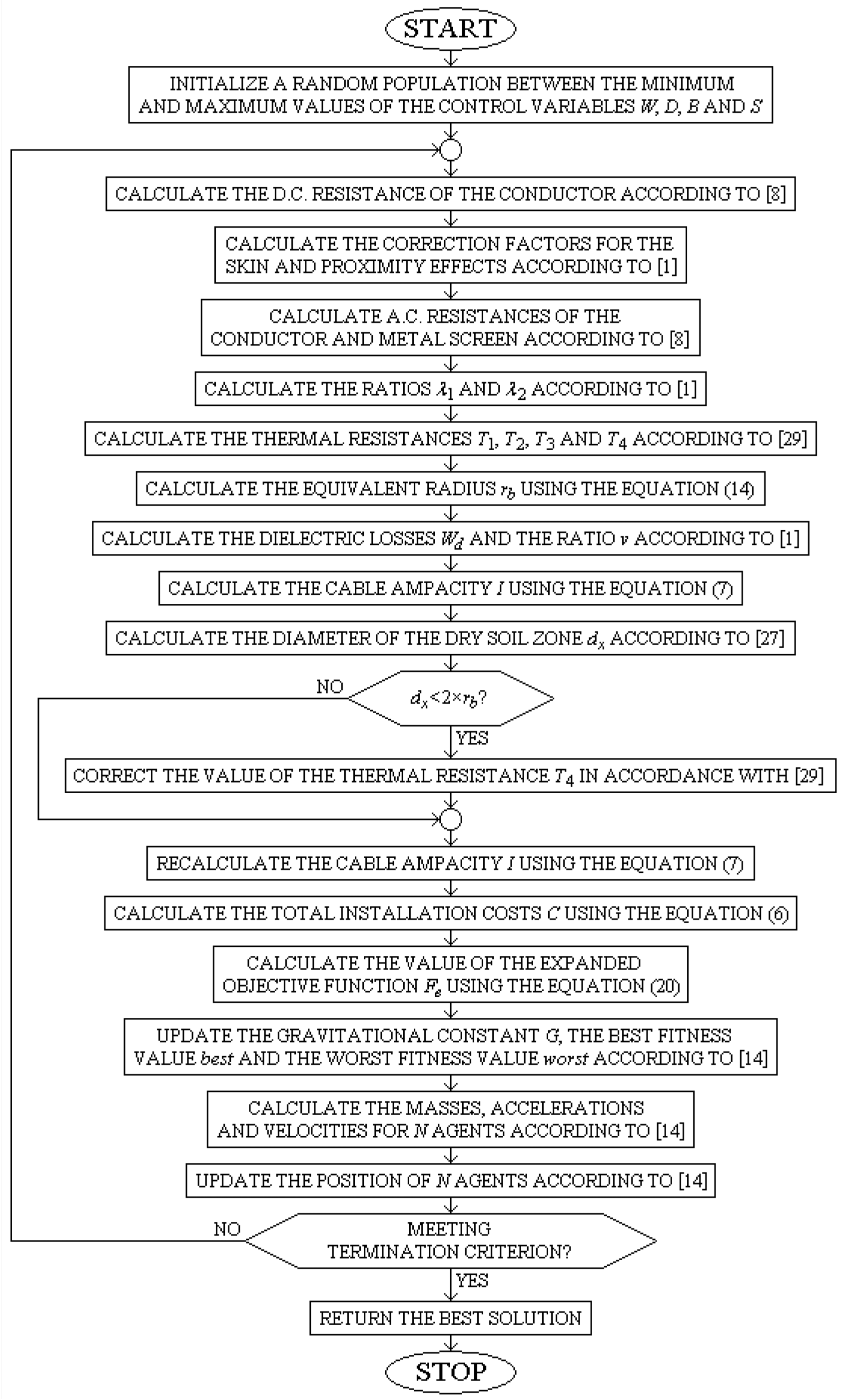

Fig. 3 *Flowchart of the GSA for optimisation of the thermal environment and the ampacity of underground power cables*

Table 3 Optimal values of variables *W*, *D*, *B*, *S* and *C* obtained by the minimisation of the total installation cost function

I, A	Optimal values					Method
	W, m	*D*, m	*B*, m	*S*, m	*C*, $/m	
310	0.3805	0.3505	0.9199	0.0403	53.4105	GSA
	0.3805	0.3536	0.9183	0.0403	53.4357	SAA
	0.4130	0.3165	0.9369	0.0565	56.0350	GPSA
360	0.4668	1.0951	0.5476	0.0834	69.4298	GSA
	0.4668	1.0951	0.5476	0.0834	69.4298	SAA
	0.6807	0.7558	0.7172	0.1878	87.2134	GPSA
410	0.7829	1.0951	0.5476	0.2415	103.7326	GSA
	0.7812	1.0951	0.5476	0.2397	103.5456	SAA
	0.8762	1.2499	0.6249	0.2881	123.4203	GPSA

Table 4 Optimal values of variables *W*, *D*, *B*, *S* and *I* obtained by the maximisation of the cable ampacity function

C, \$/m	Optimal values					Method
	W, m	*D*, m	*B*, m	*S*, m	*I*, A	
65	0.4260	1.0951	0.5476	0.0630	352.7177	GSA
	0.4308	1.0436	0.5733	0.0654	350.5297	SAA
	0.4960	0.5555	0.7611	0.0980	336.0617	GPSA
75	0.5181	1.0951	0.5476	0.1091	368.3870	GSA
	0.5181	1.0950	0.5476	0.1090	368.3523	SAA
	0.5086	1.1240	0.5620	0.1043	365.3296	GPSA
85	0.6103	1.0951	0.5476	0.1551	382.5119	GSA
	0.6103	1.0951	0.5476	0.1547	382.4872	SAA
	0.5164	1.3750	0.6875	0.1082	355.1307	GPSA

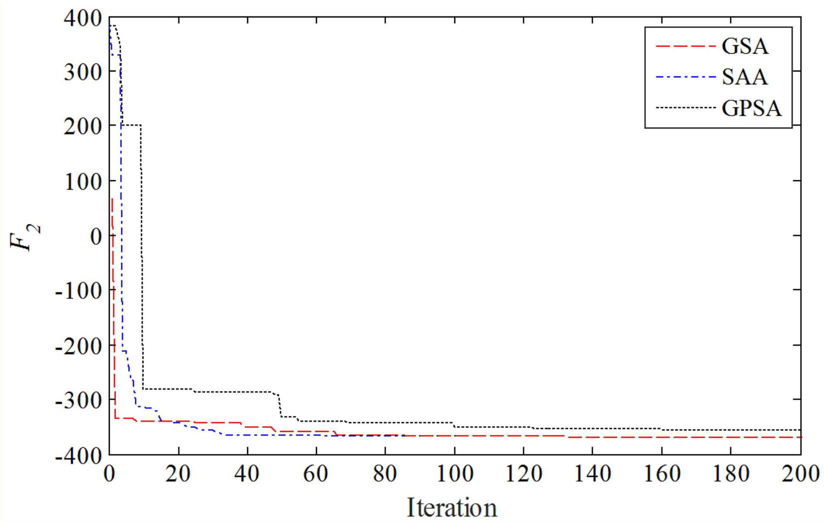

Fig. 4 *Convergence profiles of the GSA, SAA and GPSA in the case of maximisation of the cable ampacity for C = 75 \$/m*

Table 5 Optimal values of variables *W*, *D*, *B*, *S*, *C* and *I* obtained by the maximisation of the unconstrained objective function (12)

Weighting factors		Optimal values						Method
w_C, dimensionless	w_I, dimensionless	*W*, m	*D*, m	*B*, m	*S*, m	*C*, \$/m	*I*, A	
2	1	0.3806	1.0951	0.5476	0.0403	60.0690	343.2779	GSA
		0.5245	1.0883	0.5510	0.1123	75.6099	368.8491	SAA
		0.3873	0.8726	0.6587	0.0436	58.7744	334.8459	GPSA
1	1	0.9401	1.0951	0.5476	0.3140	120.7858	462.0044	GSA
		0.9947	1.1350	0.5750	0.2404	129.8640	455.9112	SAA
		1.1103	1.0701	0.5966	0.4051	140.5141	469.2924	GPSA
1	2	0.9403	1.0950	0.5476	0.3099	120.7864	463.0080	GSA
		1.0137	1.0990	0.5482	0.2186	128.9878	460.5934	SAA
		1.0625	1.2716	0.6250	0.3812	146.7458	467.8520	GPSA

ones obtained using the SAA. In addition, for the same ampacity, the GSA and SAA give approximately the same costs. In this regard, the GPSA has proven to be the worst algorithm.

There are two manners by which an increase in axial spacings between cables affects the cable ampacity. The first is the simultaneous impact of the proximity effect and 'effective' thermal resistivity between cables and the surroundings [8]. The second is the effect of ohmic losses in metal screens due to high-magnitude circulating currents [8]. In general, these two impacts, respectively, lead to potentially higher and lower ampacities. Moreover, the magnitude of these effects and the optimal axial spacing between cables depend on the cable construction, voltage level and earthing of the metal screens. On the basis of the results obtained by the GSA and shown in Table 4, it is evident that the increase of the axial spacing between cables and the increase of the bedding width raise the cable ampacity.

From the results shown in Table 5 it is clear that even when the minimisation of the total installation costs *C* is twice as important as the maximisation of the cable ampacity *I* ($w_C = 2$ and $w_I = 1$), the GSA and SAA select the bedding dimensions corresponding to the case when the trench is completely filled up. Therefore, if one aims to achieve savings in installation costs, then surely it should not be done by reducing the height of the cable bedding. It is interesting to note that for $w_C = 2$ and $w_I = 1$, all the three algorithms select the bedding dimensions similar to the ones obtained for $w_C = 1$ and $w_I = 1$. The reason for this is the fact that a large increase in the bedding dimensions would lead to a drastic increase in the total installation costs and a slight increase in the cable ampacity, which is recognised by the algorithms as uneconomical. In any case, by setting the values of the weighing factors w_C and w_I, users can choose the importance in accordance with their own abilities and needs.

In general, based on comparisons between the results given in Section 4, a standard underground installation of medium-voltage cables does not represent the best possible techno-economic solution and does not have to be applicable to all voltage levels and

Table 6 Comparison of the results obtained for optimal and traditional designs of the considered underground cable line

Optimal design *C*, \$/m	Traditional design *C*, \$/m	Percentage difference, %	Optimal design *I*, A	Traditional design *I*, A	Percentage difference, %	Repayment period [a], years	Method
Problem 1: minimisation of the total installation costs							
53.4105	66.88	+20.14	310	327.06	−5.22	[b]	GSA
53.4357		+20.10					SAA
56.0350		+16.22					GPSA
69.4298		−3.81	360		+10.07	0.422	GSA
69.4298		−3.81				0.422	SAA
87.2134		−30.40				3.37	GPSA
103.7326		−55.10	410		+25.36	2.42	GSA
103.5456		−54.82				2.42	SAA
123.4203		−84.54				3.72	GPSA
Problem 2: maximisation of the cable ampacity							
65	66.88	+2.81	352.7177	327.06	+7.84	[c]	GSA
			350.5297		+7.18		SAA
			336.0617		+2.75		GPSA
75		−12.14	368.3870		+12.64	1.07	GSA
			368.3523		+12.62	1.07	SAA
			365.3296		+11.70	1.16	GPSA
85		−27.09	382.5119		+16.95	1.78	GSA
			382.4872		+16.95	1.78	SAA
			355.1307		+8.58	3.52	GPSA
Problem 3: simultaneous application of the minimisation of the total installation costs and the maximisation of the cable ampacity							
60.0690	66.88	+10.18	343.2779	327.06	+4.96	[c]	GSA
75.6099		−13.05	368.8491		+12.78	1.14	SAA
58.7744		+12.12	334.8459		+2.38	[c]	GPSA
120.7858		−80.60	462.0044		+41.26	2.18	GSA
129.8640		−94.17	455.9112		+39.40	2.67	SAA
140.5141		−110.1	469.2924		+43.49	2.82	GPSA
120.7864		−80.60	463.0080		+41.57	2.16	GSA
128.9878		−92.86	460.5934		+40.83	2.54	SAA
146.7458		−119.42	467.8520		+43.05	3.09	GPSA

[a]The period for the repayment of the difference between the total installation costs for optimal and traditional designs of the considered underground cable line.

[b]Such a comparison is not possible since the ampacity for the optimally designed underground cable line is lower than for the traditionally designed feature.

[c]By decreasing the total installation costs, starting from the value that corresponds to the case of the traditionally designed underground cable line (i.e. from 66.88 \$/m), it is possible to increase the cable ampacity (for instance, from 327.06 to 352.7177 A etc.).

all cable constructions. Moreover, based on the value of 65 \$/m specified for the total installation costs in Table 4, the ampacity obtained by the GSA for the optimally designed underground cable line is estimated at 352.7177 A, representing a rise of 7.845% compared with the ampacity of 327.06 A for the traditionally designed feature (where $C = 66.88$ \$/m). A similar conclusion can be drawn from the results presented in the second and third rows of Table 4 (relating, respectively, to 75 and 85 \$/m), as well as the first and second rows of Table 5 (relating, respectively, to $w_C = 2$, $w_I = 1$ and $w_C = 1$, $w_I = 1$). Furthermore, according to Tables 3–5, the total installation costs and the cable ampacity can, respectively, have higher values than 66.88 \$/m and 327.06 A which are associated with the traditionally designed feature. This means, from a long-term point of view, that the optimally designed feature, which is more expensive at the beginning of the cable life span, is much better than the traditionally designed one. According to [30], the life span of medium-voltage cross-linked polyethylene (XLPE)-insulated cables amounts to 25 years.

The results obtained from the optimisation show that in most cases the total installation costs and the ampacity corresponding to the optimally designed underground cable line are higher than ones expected for the traditionally designed feature. The higher current-carrying capacity of the optimally designed feature would contribute to an additional financial benefit as well. Taking the electricity transmission tariff of 0.36 \$c/kWh into account, it is possible to determine the period in which the additional benefit could cover the difference between the total installation costs for optimal and traditional designs of the considered underground cable line. The electricity transmission tariff is selected in accordance with [31], converted in US dollars, comparable with the transmission tariffs used in [3, 32] and represents 3.2% of the end electricity tariff. As the considered transmission tariff is lower than the real one at the distribution voltage levels, it means that the optimisation procedures (i.e. the GSA, SAA and GPSA) are also safe from the engineering point of view.

The first sub-row of the second row of Table 4, relating to the GSA, shows that the total installation costs and the ampacity are equal to 75 \$/m and 368.387 A, respectively. In other words, this means that the total installation costs and the ampacity can be increased, respectively, by 8.12 \$/m and 41.327 A. Hence, the power that could be transmitted by the optimally designed underground cable line at the voltage of 35 kV is 2505.316 kW higher than the corresponding power for the traditionally designed feature (assuming that $m = 1$). Therefore, taking into account the aforementioned transmission tariff and assuming that the mean duration of the equivalent peak load $\tau_{\max}$ equals 4200 h/year, the annual financial benefit associated with the difference in the transmitted power is $2505.316 \times 4200 \times 0.0036 = 37880.38$ \$. If the considered underground cable line is 5 km in length, the difference between the total installation costs for optimal and traditional features of 8.12 \$/m will be paid back in only 1.07 years of exploitation. Such analysis is also conducted for the remaining optimisation problems. The corresponding results are given in Table 6.

There are, according to Tables 3–6, cases where the total installation costs C and the ampacity I for the optimally designed cable line are lower than the ones for the traditionally designed feature. Such pairs of C and I can only be identified as an alternative option for investors or design engineers. Of course, which pair of C and I is the right one depends on the needs and abilities of users.

Finally, it should be made clear that conductor temperature at the same ampacity is lower in the case of optimally designed underground cable line. In that respect, cables of the optimally designed underground cable line will have a longer life span than cables of the traditionally designed feature.

6 Conclusions

The conclusions that can be drawn from the results presented in this paper are as follows:

- The results obtained by means of the GSA, SAA and GPSA are in line with the defined objective functions and all the specified constraints are met.
- Although the ampacity function is highly non-linear and has a discontinuity in the transition from the case where drying-out takes place to the case where drying-out does not occur, it is demonstrated that the GSA has a very good convergence capability. This does not apply to the SAA and GPSA for the third case of optimisation.
- The height of the cable bedding should be increased in accordance with available funds, and as much as possible towards the level of the ground surface. In addition, cables should be positioned in the lower part of the bedding so that the height of the bedding-part below the cables is minimal.
- Further enhancement of the conductive heat transfer between the cables and the surroundings can be achieved by an increase in the width of the cable bedding.
- An increase in the budget leads to a significant increase in the cable ampacity, i.e. current-carrying capacity of an underground cable line. By proceeding in this manner, it becomes possible to delay or avoid investments related to the installation of an underground line consisting of cables with a greater cross-section.
- As regards the optimal solutions, the repayment periods of the total installation costs are shorter than for the traditionally designed underground cable line. The periods for the repayment are shorter than 2.42 years for the GSA and do not necessarily go in favour of the ampacity increase. It is also useful to notice that the longest repayment period which is obtained by the GSA represents 9.68% of an average life span of 25 years for medium-voltage XLPE-insulated cables. In the first two cases of optimisation, the results obtained by the SAA are nearly identical to the ones obtained by the GSA. Also, the GPSA gave the highest estimates of the repayment period.
- On the basis of the analysis conducted herein, it can be noticed that the GSA is the most stable in searching for optimal solutions. The proposed GSA can be simply and reliably applied to the selection of the optimal solution as well as to different cable arrangements, multi-circuit cable lines, load factors which are not constant with time and lower than 1 and so on.

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