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Article

# Two Interval Upper-Bound $Q$-Function Approximations with Applications

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**Abstract:** The Gaussian $Q$-function has considerable applications in numerous areas of science and engineering. However, the fact that a closed-form expression for this function does not exist encourages finding approximations or bounds of the $Q$-function. In this paper, we determine analytically two novel interval upper bound $Q$-function approximations and show that they could be used efficiently not only for the symbol error probability (SEP) estimation of transmission over Nakagami-$m$ fading channels, but also for the average symbol error probability (ASEP) evaluation for two modulation formats. Specifically, we determine analytically the composition of the upper bound $Q$-function approximations specified at disjoint intervals of the input argument values so as to provide the highest accuracy within the intervals, by utilizing the selected one of two upper bound $Q$-function approximations. We show that a further increase of the accuracy, achieved in the case with two upper-bound approximations composing the interval approximation, can be obtained by forming a composite interval approximation of the $Q$-function that assumes another extra interval and by specifying the third form for the upper-bound $Q$-function approximation. The proposed analytical approach can be considered universal and widely applicable. The results presented in the paper indicate that the proposed $Q$-function approximations outperform in terms of accuracy other well-known approximations carefully chosen for comparison purposes. This approximation can be used in numerous theoretical communication problems based on the $Q$-function calculation. In this paper, we apply it to estimate the average bit error rate (ABER), when the transmission in a Nakagami-$m$ fading channel is observed for the assumed binary phase-shift keying (BPSK) and differentially encoded quadrature phase-shift keying (DE-QPSK) modulation formats, as well as to design scalar quantization with equiprobable cells for variables from a Gaussian source.

**Keywords:** $Q$-function; approximation; Nakagami-$m$ fading; modulation formats

**MSC:** 33F05



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